Overview

Number of instruction days: 4–6

Content to Be Learned
- Explore the concept of square root.
- Evaluate square roots of small perfect squares.
- Know the meaning and the appropriate use of square root symbols.
- Identify a number as rational or irrational.
- Compare rational and irrational numbers using the number line.
- Convert a repeating decimal into a fraction.
- Show that the side length of a square is the square root of its area.

Mathematical Practices to Be Integrated
5 Use appropriate tools strategically.
- Place rational and irrational numbers on the number line.

7 Look for and make use of structure.
- Look for patterns in finding areas and side lengths for upright squares and apply that knowledge to tilted squares.

8 Look for and express regularity in repeated reasoning.
- Find patterns of rational numbers when converting repeating or terminating decimal expansions into fractional form.

Essential Questions
- What does it mean to find the square root of a number?
- What is the relationship between the side length and the area of a square?
- Why is it important to be able to approximate between which two whole numbers on a number line a given irrational number would be found?
- How are irrational numbers different from rational numbers?
Common Core State Standards for Mathematical Content

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., \( \pi^2 \)). For example, by truncating the decimal expansion of \( \sqrt{2} \), show that \( \sqrt{2} \) is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations 8.EE

Work with radicals and integer exponents.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form \( x^2 = p \) and \( x^3 = p \), where \( p \) is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that \( \sqrt{2} \) is irrational.

Common Core State Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see \( 7 \times 8 \) equals the well remembered \( 7 \times 5 + 7 \times 3 \), in preparation for learning about the distributive property. In the expression \( x^2 + 9x + 14 \), older students can see the 14 as \( 2 \times 7 \) and the 9 as \( 2 + 7 \). They recognize the
significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1), (x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In Grade 6, students found the area of special quadrilaterals in the context of solving real-world and mathematical problems. In Grade 7, students computed lengths and areas from a scale drawing. Students also solved real-world mathematical problems involving area.

Current Learning

In this unit, students understand informally that every number has a decimal representation. Students must find the fractional equivalent of a repeating decimal. Students use a number line to compare rational and irrational numbers. They also find side lengths of squares by finding the square root of the area. Students learn that a nonrepeating, nonterminating decimal is an irrational number.

Future Learning

In Algebra I, students will use properties of irrational numbers. In Algebra II they will extend this work to the complex plane.

Additional Findings

Students often struggle with converting decimals and fractions. Studies show that “the deepest translation problem in Pre-K to grade 8 concerns the translation between fractional and decimal representations of rational numbers.” (Adding It Up, p. 101)
Assessment

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

- Understand that real numbers are either rational or irrational and distinguish between the two.
- Show the decimal expansion eventually repeats for rational numbers.
- Convert a repeating decimal expansion into a rational number.
- Use a number line diagram to compare irrational numbers and approximate their location.
- Evaluate square roots of small perfect squares.
- Know that $\sqrt{2}$ is irrational.

Instruction

Learning Objectives

Students will be able to:

- Draw squares with given constraints using $5 \times 5$ dot grids and identify these squares by their areas.
- Understand the concept of square root as the side length of a square with a known area and approximate the value of the square root.
- Find the exact length of a line segment drawn on a dot grid using geometric understanding of square roots and approximate the value of the square root.
- Learn the meanings of rational and irrational number and compare and order square roots on a number line.
- Convert a repeating decimal into a fraction.
- Demonstrate understanding in new context.
Resources

Connected Mathematics 2, Pearson/Prentice Hall, 2008 Looking for Pythagoras
- Investigation 2: Squaring Off
- Student Materials (pages 19 – 30)
- Teacher’s Guide
- Implementing and Teaching Guide
- Teaching Transparencies
- Lab Sheet Resource Book
- Assessment Resource Book
- Additional Practice and Skill Workbook
- Special Needs Handbook
- Parent Guide
- Teacher CDs

Exam View
www.pearsonsuccessnet.com/snpapp/login/login.jsp

Algebra I, Glencoe, 2010
- Student Materials (pp. P7 – P10) see supplemental section of this binder
- Teacher Edition
- Online resources at http://glencoe.mcgraw-hill.com/sites/0078884802/

Note: The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery and Assessment sections for specific recommendations.

Materials
No materials required for this unit.

Instructional Considerations

Key Vocabulary
irrational number
real number

Planning for Effective Instructional Design and Delivery

Reinforced vocabulary taught in previous grades or units: rational number.

Living word walls will assist all students in developing content language. Word walls should be visible to all students, focus on the current unit’s vocabulary, and have pictures, examples, and/or diagrams to accompany the definitions.
Teachers should review the “Mathematics of the Unit” found on page 3 of all CMP2 teacher editions prior to planning. For planning considerations read through the teacher edition for suggestions about scaffolding techniques, using additional examples, and differentiated instructional guidelines as suggested by the CMP resource.

Problem 2.1 is a good place to use nonlinguistic representations to represent knowledge. Students will draw pictorial representations on dot paper to represent perfect squares and to find square roots. To launch Problem 2.1, simply explain to students that they are to find all the different possible squares and their areas that can be drawn within the confines of the 5 × 5 dot grid. Instruct students to label the interior of each square with the square’s areas. A square with the same area but different orientation (or placement) on the 5 × 5 dot grid should be considered the same square. Have students save their work from this lesson for use throughout this unit of study.

Caution: A common student error when finding side lengths is to think of the diagonal distance drawn on the centimeter dot grid as one unit in length. For example, consider the squares below that have areas of 2 and 8 square units, respectively.

Some students will mistakenly label the side lengths as 1 and 2 units, respectively, as they are counting the diagonal length as 1 unit. To help avoid this confusion, be clear in your launch of this problem by defining the horizontal unit (or vertical unit) as 1 centimeter in length. Additionally, you can guide them through this misconception by referring back to the “driving distance vs. helicopter distance” in Investigation 1. Students need to practice on both grid and dot paper because the check-up has a question using a coordinate grid.

Some students will have difficulty drawing the square on the dot grid. Have students who are successful at drawing these squares explain their strategy to those who are struggling.

Problem 2.2 examines the side lengths of the squares drawn in Problem 2.1. In the launch of Problem 2.1, the term square root is reintroduced. Students are taught that the side length of a square can be expressed as the square root of the area of the square. Consider doing parts C and D by having students explore in pairs as questions arise but then convening as a whole class to discuss their findings. For example, ask students to find the length of the square with an area of 2 units. Students may then work with a partner to explore this task. Call the class back together to share their findings and discuss. The goal in parts C and D is for students to begin to realize that while √2 and √5 have an exact length on a square, they are not able to precisely express this length as a rational number. For part E, students may record their work on their lab sheet from Problem 2.1.

At the conclusion of Problem 2.2, consider handing students a blank dot grid. Have them create a ruler/number line that is 10–12 centimeters in length and label it like a centimeter ruler (0 cm, 1 cm, 2 cm, etc.). Direct students to use their squares from Problem 2.1 and “measure” the side lengths on the number line starting at 0 cm. Students should record the actual side length of the square and label this length (e.g., √1, √2, √4, √5). They can also record the decimal approximation for these lengths on the number line. Follow up with questions such as, “If I drew a square with an area of 37 square centimeters, where would its side length fall on this number line?” Also ask students to consider the equation \( \sqrt{2} \times \sqrt{2} = ? \). Discuss the meaning of this notation and what it means to “square” a square root. ACE Questions 25–28 on page 24 of the student edition provide additional practice with this idea.
In Problem 2.3, students will apply what they have learned from this investigation to find the length of a line segment drawn on dot paper. There are 14 possible line segments that can be drawn on the 5 x 5 dot grids; however, Lab sheet 2.3 provides only 6 grids. Students will need at least 2 (or 3) copies of the lab sheet. In their groups, students do not need to find all 14 possible lengths, but as a class ensure that all 14 are presented during the summary. If students are struggling, suggest that they use their number line created in Problem 2.2 to help them reason through part A. Parts B and C informally introduce the concept of simplifying square roots. Be sure that every student is able to draw a square on a line segment, find the area of that square, and then find the length of the segment using square roots.

Assign ACE Question 41 at the conclusion of Problem 2.3. A lab sheet is provided for this particular ACE question, if needed.

Students also need to be able to evaluate expressions involving square roots. For example, determine the value of $y$ when $x = 4$ given $y = 7\sqrt{x} + 2x$. Provide a few examples in class or as warm-ups after Problem 2.2.

The Investigation 2 Additional Practice (pages 31–33 of the Additional Practice and Skills Workbook) is highly recommended for use in this unit. It can be used in class or as homework.

CMP2 has online resources that may be helpful in planning for all units of study. Visit www.phschools.com and sign on to SuccessNet.

Incorporate the Essential Questions as part of the daily lesson. Options include using them as a “do now” to activate prior knowledge of the previous day’s lesson, using them as an exit ticket by having students respond to it and post it, or hand it in as they exit the classroom, or using them as other formative assessments. Essential questions should be included in the unit assessment.

**Notes About Repeating Decimals**

The CCSS specify that students in 8th grade convert repeating decimals to fractions. The CMP2 resources do not offer any lessons on this topic. The writing teams identified appropriate material for this topic, which are listed in the Resources section. Below is a discussion quoted from the Math Forum (http://mathforum.org/dr.math/faq/faq.fractions.html):

If the decimal is a repeating decimal instead of a terminating one, we can still convert it to a fraction. Let's try to figure out the fractional equivalent of $0.5757575757$.

Let $F$ be this fraction. We see that the repeating group has length 2. This tells us to multiply $F$ by $10^2 = 100$. See what happens when we do:

\[ F = 0.5757575757... \]
\[ 100F = 57.5757575757... \]

Now we can subtract the first equation from the second, and the repeating part of the decimal is cancelled:

\[ 99F = 57.0000000000... = 57 \]

(Note you can see why we chose $10^2$ as a multiplier. It was just to make this cancellation happen.)

Now it is easy to find $F$. Remember to reduce it to lowest terms when you have it in the form of a fraction:

\[ F = \frac{57}{99} = \frac{19}{33}. \]
Sure enough, when we do long division, we find that $19/33 = 0.5757575757$.

As another example, let's convert $F = 1.3481481481481...$ to a fraction. Since the repeating group has length 3, we should multiply $F$ by $10^3 = 1000$.

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F = 1.3481481481481...
$$
$$
1000F = 1348.1481481481481...
$$
$$
999F = 1346.8000000000000...
$$
$$
= 1346.8 = 6734/5,
$$
$$
F = 6734/(5*999) = 6734/4995 = 182/135
$$