Algebra I, Quarter 2, Unit 2.4

Applications of Systems of Equations and Inequalities

Overview

Number of instruction days: 8–10

(1 day = 53 minutes)

Content to Be Learned

- Prove that, given a system of two equations in two variables, replacing one equation with the sum of that equation and a multiple of the other produces a system with the same solutions.
- Solve systems of two equations in two variables exactly and approximately.
- Solve a system of equations consisting of a linear equation and a quadratic equation in two variables.
- Explain why the intersection of two functions graphed on a coordinate plane ($y = f(x)$ and $y = g(x)$) is the solution to the equation $f(x) = g(x)$.
- Graph the solution set to a system of linear inequalities in two variables as the intersection of two half-planes.
- Represent constraints with equations and/or systems of equations.
- Interpret solutions as viable or not viable in a modeling context.

Essential Questions

- How do you interpret the intersection of two graphs in the context of a problem?
- How do you determine the best method for solving a system of equations?
- How do you know if a solution is reasonable?
Standards

Common Core State Standards for Mathematical Content

Algebra

### Reasoning with Equations and Inequalities A-REI

#### Solve systems of equations

- **A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **A-REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### Represent and solve equations and inequalities graphically

- **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
- **A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

### Creating Equations* A-CED

#### Create equations that describe numbers or relationships

- **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Clarifying the Standards

Prior Learning

Students learned to graph points on the coordinate plane and interpreted the values of coordinates in the context of the situation in Grade 5. In Grade 6, students learned the process of finding a solution set for an equation, and they solved equations of the form $x + p = q$ and $px = q$. They learned to write inequalities and represent solutions to a simple inequality on a number line. They also learned to write equations to describe relationships between two quantities. In Grade 7, students used the properties of operations to generate equivalent linear expressions. They also learned to use algebraic equations and inequalities to solve word problems. In Grade 8, students solved linear equations in two variables, including equations with 1, 0, and infinitely many solutions, and equations in which they applied the distributive property and collected like terms. Students began to develop techniques to solve and analyze systems of equations algebraically and graphically. They learned that the intersection of the graphs of two linear equations represented the solution to a system of equations. Finally, students learned the definition of a function, compared functions represented two different ways, and wrote functions to model linear relationships. They also examined and explained functions that were not linear.

Current Learning

Students are expected to be fluent with the use of systems of linear equations and inequalities in two variables. In this unit, students solve systems of equations exactly and approximately after proving different methods for solving systems. Students represent and solve systems of equations and inequalities. They explain why the $x$-coordinates of the intersection points of two graphs are the solutions to the equation. Students use technology to graph two functions, make tables of values, and find successive approximations. Students graph the solution to a system of linear inequalities as the intersection of two half-planes. They represent constraints using equations, inequalities, and systems of equations, and they interpret solutions as viable or nonviable options in a modeling context.

Future Learning

In Algebra II, students will create equations and represent constraints with equations and inequalities using a variety of function types including linear-quadratic. They will graph and analyze a variety of function types, including radical, rational, polynomial, and trigonometric functions. They will also represent and solve systems of equations graphically, using a number of different function types. Further study of systems of equations will occur in Precalculus, where students will represent a system of linear equations as a matrix equation on a vector. They will also use the inverse of a matrix to solve systems of linear equations, using technology for larger systems. In advanced mathematics courses, including linear algebra and differential equations, students will represent systems of multiple equations with matrices. Systems of equations, both linear and nonlinear, will be essential for student success in advanced courses in physics, economics, and chemistry. Representing and solving equations and inequalities graphically is a major standard in Algebra I according to the PARCC Model Content Frameworks.

Additional Findings

In A Research Companion to Principles and Standards for School Mathematics, Chazan and Yerushalmy discuss the cognitive difficulties that many students have in working with the complex relationships embedded in systems of equations. As an example, they describe the methods that students must use to solve a system of equations consisting of a linear equation in standard form and a circle in standard form. As students work through the solving of such a system, they must move “from an equation in two variables to a function of one” to enable use of the substitution algorithm “from an equation in two
variables to an equation in one variable” using the algorithm, to generating equivalent expressions in solving the new equation. Chazan and Yerushalmy indicate that this complexity is common in learning about equivalence in school algebra, and that this cognitive complexity must be taken into account when approaching topics involving equivalence (pp.129-131). Chazan and Yerushalmy also indicate that graphing technology can assist students in making sense of equivalent expressions. (p. 130)

**Assessment**

When constructing an end-of-unit assessment, be aware that the assessment should measure your students’ understanding of the big ideas indicated within the standards. The CCSS for Mathematical Content and the CCSS for Mathematical Practice should be considered when designing assessments. Standards-based mathematics assessment items should vary in difficulty, content, and type. The assessment should comprise a mix of items, which could include multiple choice items, short and extended response items, and performance-based tasks. When creating your assessment, you should be mindful when an item could be differentiated to address the needs of students in your class.

The mathematical concepts below are not a prioritized list of assessment items, and your assessment is not limited to these concepts. However, care should be given to assess the skills the students have developed within this unit. The assessment should provide you with credible evidence as to your students’ attainment of the mathematics within the unit.

- Solve systems of two equations in two variables using various methods.
- Model systems of equations from real-world situations.
- Determine the best method for solving systems of equations in real a real world context.
- Interpret the intersection of two graphs in a real world context.
- Explain why the intersection of two functions graphed on a coordinate plane is the solution to that system.
- Solve a system of linear inequalities in two variables.

**Instruction**

**Learning Objectives**

Students will be able to:

- Solve systems of linear equations by graphing.
- Solve systems of equations using substitution.
- Solve systems of equations using elimination with addition or subtraction.
- Solve systems of equations using elimination with multiplication.
- Determine the best method for solving systems of equations.
- Solve systems of linear inequalities by graphing.
- Demonstrate understanding of concepts and skills learned in this unit.
Resources

*Algebra*, (Glencoe McGraw Hill) 2010

- Section 6.1 pp. 333 to 339
- Section 6.1 Extend pp. 340 to 341
- Section 6.2 pp. 342 to 347
- Section 6.3 pp. 348 to 354
- Section 6.4 pp. 355 to 360
- Section 6.5 pp. 362 to 367
- Section 6.8 pp. 382 to 386
- Section 6.8 Extend p. 387
- *Chapter 6 Resource Masters* of corresponding sections
- *5-Minute Check Transparencies* for corresponding sections

*Real-World Problem Solving* (Graphic Novel), (Glencoe McGraw-Hill) 2010

- Section on Systems of Equations (pages 13-18)


- Interactive Classroom CD - PowerPoint presentations (optional)
- Teacher Works Plus CD-ROM
- Teaching with Foldables (Dinah Zike; Glencoe McGraw Hill 2010)

Exam View Assessment Suite Software

Math Online, glencoe.com

*Note:* The district resources may contain content that goes beyond the standards addressed in this unit. See the Planning for Effective Instructional Design and Delivery and Assessment sections for specific recommendations.

Materials

Graphing calculators, grid paper, colored pencils, algebra tiles, PowerPoint presentation (optional)
Instructional Considerations

**Key Vocabulary**

<table>
<thead>
<tr>
<th>Consistent</th>
<th>Independent</th>
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<tbody>
<tr>
<td>Dependent</td>
<td>Point of intersection</td>
</tr>
<tr>
<td>Elimination</td>
<td>System of equations</td>
</tr>
<tr>
<td>Inconsistent</td>
<td>System of inequalities</td>
</tr>
</tbody>
</table>

**Planning for Effective Instructional Design and Delivery**

Reinforced vocabulary taught in previous grades or units: substitution, equation.

Living word walls will assist all students in developing content language. Word walls should be visible to all students, focus on the current unit’s vocabulary, both new and reinforced, and have pictures, examples, and/or diagrams to accompany the definitions.

You may provide students with graphic organizers. One example would be using an organizer to assist students with identifying similarities and differences by comparing the different methods of solving a system of equations. Identifying the best method to use when solving a particular system of equations is a critical skill in this unit.

On page 332, Foldables are referenced for students to use in making study guides from notes, examples, graphs, or other representations regarding systems of equations. They can also serve as an additional tool for identifying similarities and differences through comparing the different methods for solving a system, thereby organizing their knowledge. Foldables also offer students a sense of ownership in their work and learning.

For Section 6-1, consider using the Graphing Calculator Activity on page 11 of the Chapter 6 resource masters as a follow-up to using the graphing strategy for solving systems of equations. Use this activity as an introduction to the unit.

Have students refer to the graphic novels provided in the ancillary materials regarding systems of equations. The alternate format for presenting problem solving can be very helpful for students needing a more visual format of information.

Have students work in pairs for graphing systems of equations, taking turns between graphing the equation and checking the solution set. Discuss with students when a system of equations has one solution, no solution, or infinitely many solutions. Write examples of systems on the board and have students answer: one, none, or infinite solutions.

In the classroom, have a blog where students can write entries, explaining how they decide when to use each of the different methods for solving systems of equations. One thing they can do is write a pros and cons list for each method. See Study Notebook Chapter 6 graphic organizer (p. 111 in the Teacher Edition).

If algebra tiles are available, have students use these physical models (nonlinguistic representations) to solve systems of equations using elimination. Problem 33 on page 353 is a good place for students to use physical models while elaborating on their knowledge.
When students begin to use elimination using multiplication, they sometimes forget to multiply each term on both sides of the equation. You may ask students to show this step when they are learning to apply it to help them remember to multiply each term. If not, be sure to ask questions about this step to ensure that students understand the method.

The supplementary resource for **CCSS Graphing Technology Lab 13: Solving Exponential Equations and Inequalities** is accessible online on the Glencoe McGraw-Hill Math Algebra Student and Teacher textbooks. First, select the CCSS icon on the homepage of the online textbook, select the CCSS Supplement tab, and then select the respective lab or lesson. The lesson is also located in the supplemental section of the curriculum binder.

Graphing technology will assist students with modeling real-world problems involving systems of equations and inequalities and for identifying the solution to a system of equations.

For planning considerations read through the teacher edition for suggestions about scaffolding techniques, using additional examples, and differentiated instructional guidelines as suggested by the Glencoe resource.

Incorporate the Essential Questions as part of the daily lesson. Options include using them as a “do now” to activate prior knowledge of the previous day’s lesson, using them as an exit ticket by having students respond to it and post it, or hand it in as they exit the classroom, or using them as other formative assessments. Essential questions should be included in the unit assessment.

The 5-minute check transparencies can be used as a **cue, questions, and advance organizers** strategy as students will be activating prior knowledge. Some 5-minute checks may take longer than the allotted time, so consider choosing only problems that activate prior knowledge and use the rest for differentiation, to formatively assess student learning, as an exit ticket, or assigning for homework.

You may use the transparencies provided in the ancillary material as focus activities, review, or an exit activity. Additional activities and examples may be used for homework assignments.